Mathematical description of a technique for standardized evaluation of any signal encountered in personal safety applications involving low-frequency electric or magnetic fields

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Introduction

Existing personal safety standards and guidelines for low-frequency electric and magnetic fields (f < 100 kHz) contain reference values for the electric field strength and magnetic flux density that vary as a function of frequency. These reference values have been derived, based on the tolerance of the human body, from the biological limits for current densities. In other words, if field measurements at a given site yield values below the reference values, then the presence of persons at the site is permitted.

The reference values are straightforward when applied to stationary sinusoidal signals. However, proper evaluation of pulsed or composite signals requires access to additional information from the relevant standards and guidelines. Real-world experience has shown that this “additional information” is often ambiguous and subject to varying interpretations by different persons.

In this paper we will present a technique which, while not in conflict with existing standards and guidelines, extends them through a mathematical description which enables unambiguous evaluation of any signal.

Reference values and evaluation filters

The standards and guidelines mentioned above contain frequency-dependent reference values in the form of tables. In these tables, the different line segments are presented using a double logarithmic scale. In the low-frequency range, the slope of the line segments is 0, -20 dB or -40 dB per decade.

The reference values can be mathematically approximated very precisely using Eq. 1.

$$G_{ref}(f) = G_0 \cdot \sqrt{\frac{1+\frac{f^2}{f_{z1}^2}}{1+\frac{f^2}{f_{n1}^2}} \cdot \frac{1+\frac{f^2}{f_{z2}^2}}{1+\frac{f^2}{f_{n2}^2}} \cdot \frac{1+\frac{f^2}{f_{z3}^2}}{1+\frac{f^2}{f_{n3}^2}}} \qquad \text{Eq. 1}$$

The limit frequencies f_z and f_n are obtained from the intersection points of the line segments.

Quite often due to rounding errors in the tables, the intersection points do not correspond exactly to a frequency given in the tables. For many of the reference curves, the first term is sufficient, while others require two terms. More than three terms are never required in the frequency range to 100 kHz. The value G_0 represents the reference value at the lowest limit frequency.

The reference curves from the tables and the reference curves based on Eq. 1 deviate from one another noticeably only at the intersection points of the lines. The reference curves based on Eq. 1 no doubt characterize the physical and biological effects better than the disjointed reference curves obtained directly from the tables. For precise standardized evaluation, we should thus use the reference values from Eq. 1.

If the individual field functions $g_x(t)$, $g_y(t)$ and $g_z(t)$ representing the field to be evaluated are available, the functions $n_x(t)$, $n_y(t)$ and $n_z(t)$, which are normalized to the reference curve, can be obtained through convolution of the field functions with the impulse response $h(t)$ of three identical evaluation filters. The magnitude of the transfer function $H(f)$ of these evaluation filters must exactly equal the reciprocal of $G_{ref}(f)$. This condition is fulfilled by Eq. 3.

$$\left|H(f)\right| = \frac{1}{G_{ref}(f)} \qquad \text{Eq. 2}$$

$$H(f) = \frac{1}{G_0} \cdot \frac{1 + j \cdot \frac{f}{f_{n1}}}{1 + j \cdot \frac{f}{f_{z1}}} \cdot \frac{1 + j \cdot \frac{f}{f_{n2}}}{1 + j \cdot \frac{f}{f_{z2}}} \cdot \frac{1 + j \cdot \frac{f}{f_{n3}}}{1 + j \cdot \frac{f}{f_{z3}}} \qquad \text{Eq. 3}$$

Eq. 3 describes the transfer function of a filter having filter elements of the 1^{st} order connected in series.

The spectra of the normalized signals correspond exactly to the spectra of the field function spectra normalized to the reference values.

$$n(t) = g(t) * h(t) \leftrightarrow N(f) = G(f) \cdot H(f) \qquad \text{Eq. 4}$$

Evaluation using different detector types

Examination of existing standards and guidelines makes it clear that for signals that are more complex than stationary sinusoidal signals, both detector types (RMS and peak) are necessary. Many of the standards only require a single type, but in some cases both types are needed simultaneously. The following technique can be used with all of the existing standards and guidelines:

First, we compute the instantaneous square of the absolute values of the three normalized time functions according to Eq. 5.

$$m(t) = [n_x(t)]^2 + [n_y(t)]^2 + [n_z(t)]^2 \qquad \text{Eq. 5}$$

Then, we compute the square of the RMS and peak value according to Eqs. 6 and 7.

$$m_{RMS}(t) = \frac{1}{T_{RMS}} \cdot \int_{t-T_{RMS}}^{t} m(x) \cdot dx \qquad \text{Eq. 6}$$

$$m_{Peak}(t) = \text{Maximum}(\text{m(x)}) \quad \text{where } x = t - T_{Peak} \ \ldots \ x = t \qquad \text{Eq. 7}$$

In existing standards in which the RMS value is important, it is stipulated that $T_{RMS} = 1\,\text{s}$.

(Note: It is entirely sufficient to compute both values at a separation of $\frac{T_{RMS}}{4}$. The holding time for the peak values T_{Peak} has to be at least as large as this separation.)

Following multiplication by an evaluation factor of k_{RMS} or k_{Peak}, we choose the larger value of the two, take the square root and multiply by 100%. The result is the instantaneous exposure $q(t)$ as a percentage of the reference value.

$$q(t) = \sqrt{\text{Maximum}(k_{RMS} \cdot m_{RMS}(t), k_{Peak} \cdot m_{Peak}(t))} \cdot 100\% \qquad \text{Eq. 8}$$

According to existing standards and guidelines, k_{RMS} is either 0 or 1 and k_{Peak} lies between 0 and 0.5.

Implementation using digital filters

The evaluation filters can be easily implemented using analog filters with the limit frequencies described above. However, in many cases it makes sense to use digital filters. Here, we have to compute the filter coefficients for the digital filters using a bilinear transformation. We obtain the best accuracy by individually transforming the filter elements while ensuring that the frequency corresponding to the geometric mean of f_z and f_n is represented at the same position through the transformation. This ensures that the asymptotes of the analog and digital filters are identical.