

## Uncertainty in the measurement of electromagnetic field with isotropic broadband sensor and selective E&H field analyzer

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*(E&H selective field analyzer note based on model NARDA EHP-200, frequency range 9kHz-30MHz)*

### ABSTRACT

The evaluation of uncertainties in the measurement of electromagnetic field requires a certain mastery of basic statistical concepts and an adequate understanding of the methods of measurement as well as the instrumentation used. The current methodology can be a generic way of analyzing the input variables and thus leaving very uncertain value measured. This uncertainty, however, is reduced by adopting a methodological approach which limits the assessment to a certain spectrum of frequencies, knowing the specific frequencies under examination and to a certain range of input variables of the mathematical model. To do this it is necessary to increase the information you have on the environment and the measurement system in an appropriate way using the technical specifications and calibration factor that characterize the electromagnetic field sensors. The aim of this work is to introduce some fundamental metrological concepts and exemplify some real situations in the calculation of the uncertainties associated with measurements of electromagnetic fields in high-frequency performed with isotropic broadband sensor and selective field analyzers.

### I. BASIC CONCEPTS FOR THE ANALYSIS OF UNCERTAINTY

Without pretense of rigor and completeness, below you can find some concepts and basic metrological terms.

**TRUE VALUE:** it is assumed that the quantity to be measured has a value in itself (*an sich*), that the measure approximates more or less well. Sometimes we speak of "conventional true value"

**ERROR AND UNCERTAINTY:** is also quite common among engineers in the sector that the two words "mistake" and "uncertainty" are used with a certain aplomb, almost interchangeable. In reality the two concepts are different. The difference between the measure and the "true value" is commonly called error. This error can be detected only after it has come to an estimate of the "true value". Once made this estimate and associated with it an "uncertain", the error can be corrected while the 'uncertainty', which indicates the magnitude of what has remained uncertain or has not managed to learn the "true value" or extent of the mistakes, will not be editable.

**RESOLUTION:** is the ability of instruments to distinguish between two close but different values, of the measurand

**SENSITIVITY:** It is the minimum amount that can be distinguished from zero with the instrument in use. It is a feature of the instrumentation and method of measurement used.

**REPEATABILITY:** it is a measure of which can have between subsequent measurements of the same measurand obtained by the same operator with the same tools under the same instruments and the same procedure, in the same location, in a short distance of time. It is clear from this list that we want to exclude from this the effects of changes in systematic effects. The repeatability is then connected with the sole source of random uncertainty [2].

**ACCURACY :** is a measure of uncertainty arising from imperfect knowledge of systematic errors.

**PRECISION:** is the degree of compactness of the measures with each other when it comes to repeated measures of the same quantity. Not to be confused with accuracy.

## II. ESTIMATION OF THE UNCERTAINTY OF A MEASURE

To represent accurately and completely a result of measurement is needed to give unambiguous indications of the estimate of the measurand, of the uncertainty of measurement and of the "confidence" that has it.

To measure the value of a certain size it is required a "physical model" of the measurement system, and above all, by adopting the ISO Guide [1], to know the functional relationship which exists between the measurand  $Y$  to a number  $N$  of other quantities  $X_i$ , to which measurand is related to.

Even in simple cases such as those that will be further analyzed, in which an electric field meter returns "directly" the value of a magnitude, the measurement is indirect that means that what is observed is actually the value of a magnitude  $X$  downstream the instrument, which depends on the size of the  $Y$  value that drives up the instrument and the characteristics of it.

Referring to a more thorough and detailed assessment of the basic texts [3] or the ISO Guide [1], we can state that the estimate  $y$  of the measurand can be expressed according to the report  $y = f(x_1, x_2, \dots, x_N)$  where  $x_i$  is the set of observations or estimates associated with the values of the quantity  $X_i$ .

Knowing the variances of the estimates of input  $X_i$  is possible to derive the variance of the estimate of output in accordance with the so-called law of propagation of uncertainty which is preferably applicable (see ISO Guide [1]),

for not correlated input variables, in the form  $u_c^2(y) = \sum_{i=1}^N \left( \frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)$  where  $u(x_i)$  are defined standard uncertainty

of entry estimates, the  $(\partial f / \partial x_i)$  are called sensitivity coefficients, almost always indicated as  $c_i$  and  $u_c(y)$  is defined as combined standard uncertainty and is an estimated standard deviation and characterizes the dispersion of value that could reasonably be attributed to the measurand.

Practically, the sensitivity coefficients describe how a change in the estimation of input  $x_i$  influence the estimate of output  $y_i$ , so how much it is "sensitive" to input estimate. They are therefore weights, in statistical sense, which are attributed to standard uncertainties of inputs to develop the standard combined uncertainty.

Standard input uncertainties are essentially divided into two categories, the **standard uncertainty type A** and the **standard uncertainty of type B** [1].

In the first case we are dealing with a statistical sample, coming from a series of observations of the magnitude measured, to which are applicable normal tools of statistics to obtain the best estimate of the input quantity and its variance.

In the second case we did not have a statistical sample, but only one data coming from an observation, or from an external source to the measurement (as instrument specifications or calibration certificates) or an opinion of the operator, as in the case of a poorly known systematic effect of which his entity is estimated in order to make the necessary correction. The evaluation of the associated variance, in that case, is not done with the usual statistical methods.

It should be noted that to the standard input uncertainties have to be associated a degree of freedom indicated as  $\nu$ . Without going into detail, we note that, intuitively, a statistic is more reliable larger is the sample on which it is calculated. Therefore the degree of freedom  $\nu$  are nothing more than a quantitative expression of reliability of a statistic. For the standard uncertainty type A the degree of freedom are given by the number of measurements minus one. For the standard uncertainty B usually is assumed that their evaluation tend to be conservative, so these types of uncertainty have high degrees of freedom, even tending to infinity for input estimates with a rectangular probability distribution.

Prior to exemplify an estimate of uncertainty in the measurement of electromagnetic fields it is necessary to discuss about expanded uncertainties and confidence intervals.

To the combined standard uncertainty associated with to the measure is generally associated (ISO Guide requires it) the specification of an interval (confidence interval) that includes a significant portion, and if possible known, of the values attributed to the measurand. The quantitative assessment of uncertainty which meets this requirement is the expanded uncertainty, indicated by  $U(y)$ , obtained by multiplying the combined uncertainty with an appropriate factor  $k$ , named coverage factor. Normally, the coverage factor is between 2 and 3.

The determination of a confidence interval that pertain to a probability distribution, consists in identifying two limits within which it can be said that the value of the measurand falls with a specified probability of coverage or level of confidence. The difficulty of this evaluation is that we do not know the exact distribution of the output estimate but only its estimate.

The Welch-Satterthwaite formula [1], which gives an approximate answer, gives the actual degree of freedom  $\nu_{eff}$  of the standard combined uncertainty according to the degrees of freedom and the standard uncertainty of input estimates. Without going into depth here out of place, we say that smaller are freedom degrees, greater is the coverage factor  $k_p$ , suitable to obtain a confidence interval with coverage probability  $p$ . When the degrees of freedom tend to infinity,  $k_p$  tend to a value defined for a normal distribution. Practically with  $\nu_{eff} > 20$ , with a coefficient  $k_p$  of about 2, we will obtain a confidence interval with coverage probability of 95%.

To better assess the uncertainties it is possible to divide the uncertainty budget into various methods of analysis in order to move from a high, but conservative, value of uncertainty, which leaves much doubt regarding the measured value, to a reasonably reliable value where the assessment is reduced to a certain frequency spectrum and to a certain input variables interval.

### III. ESTIMATE OF UNCERTAINTIES IN THE MEASUREMENT OF ELECTRIC FIELDS IN HIGH FREQUENCY WITH ISOTROPIC BROADBAND METERS

Uncertainty estimation that we are going to consider are related to measures of the intensity of the electric component of an electromagnetic field performed with a high frequency field sensor. To evaluate the uncertainty measurement according to the ISO Guide is necessary, in general, a descriptive model. In our case the measure is directly detectable by the from the field meter that shows the intensity value. This measure, as mentioned previously, is to be considered as "indirect" and the evaluation of the measurement uncertainty can be made from input estimates, mainly in strict relation to the instrument specifications and to the associated standard uncertainties.

Below are described the estimation of the different contributors that contribute to the measurement uncertainty budget considering the widest reasonable ranges of input variables of the measuring system.

#### - *Uncertainty of the calibration of the sensor* $u(CF)$

In the calibration phase, the sensor is immersed in a known value of electric field. At this value is obviously associated an uncertainty depending strictly on the calibration chain: power meters, generation antennas, anechoic chamber, TEM cells, etc.. These levels of uncertainty are the "best measurement capability" of the laboratory (such as for example the values indicated in the Accreditation Table of SIT Center n° 008 Narda STS S.r.l.) and they vary depending on the calibration level and frequency.

At this value should be added the uncertainty due to repeatability of the measurements of the sensor under calibration

SIT centers or equivalent report in the certificate, for each single measure, the respective value of expanded uncertainty. For example, reading the SIT certificate, the expanded uncertainty, worst case, was 0.16 (16%). The probability distribution is considered to be Gaussian and therefore gives a relative standard uncertainty of 8.0%.

#### - *Uncertainty of the frequency response of the sensor* $u(F)$

Generally, technical specifications of the field sensor are provided with a range of operating frequency and a flatness in dB or as a percentage within the frequency band. This information is essential to assess the measurement uncertainty in situations where emission frequencies of the electromagnetic field are unknown or where there are multi-frequency emission. In this case is not possible to correct the measure through the calibration factor supplied with the certificate of the sensor, but is possible to consider only the worst flatness in frequency and utilize this data as a contributor to the measurement uncertainty.

In case of technical specification of 1,2 dB, with a rectangular probability distribution, the result is a relative standard uncertainty of 8.6%

- *Uncertainty of the non-linearity*  $u(L)$

From the data provided by the technical specifications and/or by the calibration certificate, is possible to consider an uncertainty due to the non-linearity.

This assumption is valid if are considered "true values" of the field intensity not near to the sensitivity of the instrument. The probability distribution is considered to be rectangular. Whereas the technical specifications of 0.5 dB, the relative standard uncertainty is of 3.4 %.

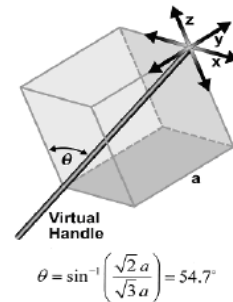
- *Uncertainty of the anisotropy*  $u(A)$

Anisotropy (A) is defined as the maximum deviation from the geometric mean of maximum and minimum value when the sensor is rotated around the ortho-axis (e.g., probe handle, rigid or flexible feed-line assembly, "virtual handle") as described in the standard IEEE std.1309<sup>TM</sup> 2005 (see below).

Whereas the technical specifications of the sensor an anisotropy of 0.8 dB, the relative standard uncertainty is of 5.6 %. The probability distribution is considered to be rectangular.

$$A = 20 \cdot \log_{10} \left( \frac{S_{\max}}{\sqrt{S_{\max} \cdot S_{\min}}} \right) dB$$

where S is the measured amplitude in the field strength units.



- *Uncertainty of the resolution of the measurement system*  $u(Res)$

In case of technical specification of 0.01 V/m and a detectable intensity field of 0.2 V/m, the relative standard uncertainty due to resolution is estimate assuming a rectangular probability distribution with semi-amplitude equal to the half of the resolution, obviously normalized at the value indicated by the meter. The result is a relative standard uncertainty of 1.4 %

- *Uncertainty of the temperature variation*  $u(Temp)$

The technical specification of the sensor report highest deviation related to the temperature in a given range or the deviation due to 1 degree Celsius variant of the temperature referred to the environment temperature (usually 23 degree Celsius).

If a deviation of 0.02 dB/°C normalized at a temperature of 23 degree Celsius is taken into account, it is necessary to consider the maximum temperature range of the measurement.

Supposed a temperature range from 10 to 40 °C the maximum variation from 23 °C is 17°C which corresponds to a deviation of 0.34 dB max.

Considering a rectangular probability distribution, the result is a relative standard uncertainty of 2.3%

- Uncertainty of the repeatability of the measures  $u(R_{mis})$

When a measurement is performed, is necessary a estimation of its repeatability. It is possible to calculate it starting from the value of the variances of the measurements done. Having collected the experimental data it shall be evaluated:

- 1) The mean value of the readings (where N is the number of readings)

$$E_{meas} = \frac{1}{N} \sum_{i=1}^N E_{meas\ i}$$

- 2) The standard deviation ( $1\sigma$ ):

$$u(E_{meas}) = \sqrt{\frac{1}{N(N-1)} \sum_{i=1}^N (E_{meas\ i} - E_{meas})^2}$$

- 3) The relative value  $\frac{u(E_{meas})}{E_{meas}}$

For our purposes we assume that were done 5 measures and was achieved a standard uncertainty of 2%.

The uncertainty balance is summarized in the table below. Since all the input contributors can be considered unrelated, the value of the combined uncertainty is obtained by adding as root mean square the individual uncertainty contributors.

Table III.1: Combined standard uncertainty – generic example of broadband sensor

Quantity	Relative standard Uncertainty in (%)	Probability distribution	Sensitivity coefficient	Degrees of freedom	Relative uncertainty contribution
CF	8.0	normal	1	$\infty$	0.08
F	8.6	rectangular	1	$\infty$	0.086
L	3.4	rectangular	1	$\infty$	0.034
A	5.6	rectangular	1	$\infty$	0.056
Res	1.4	rectangular	1	$\infty$	0.014
Temp	2.3	rectangular	1	$\infty$	0.023
$R_{mis}$	2.0	normal	1	4	0.02
$U_c$	Combined standard uncertainty	normal		$\infty$	0.139

Associated expanded uncertainty U is obtained by multiplying the total uncertainty for a coverage factor  $k = 2$  which corresponds to a confidence level of 95% as recommended by international standards. In the above mentioned case we obtain a value of 27.8%

#### IV. ESTIMATE OF UNCERTAINTIES IN THE MEASUREMENT OF ELECTRIC AND MAGNETIC FIELDS WITH EHP200 FIELD ANALYZER

Uncertainty estimation that we are going to consider are related to measures of the intensity of the electric and magnetic field performed with a EHP200 field analyzer.

Considering that EHP200 is a selective analyzer, it is possible to divide the uncertainty analysis in different frequency ranges or in specific single frequencies.

In the following table are reported the typical value of relative standard uncertainty, considering a reasonable ranges of input variables of the measuring system with a certain frequency spectrum e/o single frequency

##### - EHP200 Magnetic field – Mode A

**Table IV.1 - Frequency range 9kHz- 30kHz <sup>(1)</sup> - Typical value of uncertainty**

Quantity	Relative standard uncertainty in (%)	Probability distribution	Sensitivity coefficient	Degrees of freedom	Relative uncertainty contribution	
CF	3.9 <sup>(2)</sup>	normal	1	∞	0.039	
F	8.6	rectangular	1	∞	0.086	
L	3.4 <sup>(3)</sup>	rectangular	1	∞	0.034	
A	5.2	rectangular	1	∞	0.052	
Res	1.0 <sup>(4)</sup>	rectangular	1	∞	0.01	
Temp	2.3 <sup>(5)</sup>	rectangular	1	∞	0.023	
R <sub>mis</sub>	2.0 <sup>(6)</sup>	normal	1	4	0.02	
U <sub>c</sub>	Combined standard uncertainty	normal		∞	0.117	
					expanded uncertainty (k=2)	23.4%

**Table IV.2 - Frequency range 30kHz- 150kHz <sup>(1)</sup> - Typical value of uncertainty**

Quantity	Relative standard uncertainty in (%)	Probability distribution	Sensitivity coefficient	Degrees of freedom	Relative uncertainty contribution	
CF	3.9 <sup>(2)</sup>	normal	1	∞	0.039	
F	2.7	rectangular	1	∞	0.027	
L	3.4 <sup>(3)</sup>	rectangular	1	∞	0.034	
A	4.5	rectangular	1	∞	0.045	
Res	1.0 <sup>(4)</sup>	rectangular	1	∞	0.01	
Temp	2.3 <sup>(5)</sup>	rectangular	1	∞	0.023	
R <sub>mis</sub>	2.0 <sup>(6)</sup>	normal	1	4	0.02	
U <sub>c</sub>	Combined standard uncertainty	normal		∞	0.08	
					expanded uncertainty (k=2)	16.0%

**Table IV.3 - Frequency range 150kHz- 3MHz <sup>(1)</sup> - Typical value of uncertainty**

Quantity	Relative standard uncertainty in (%)	Probability distribution	Sensitivity coefficient	Degrees of freedom	Relative uncertainty contribution	
CF	3.9 <sup>(2)</sup>	normal	1	∞	0.039	
F	2.0	rectangular	1	∞	0.02	
L	1.9 <sup>(3)</sup>	rectangular	1	∞	0.019	
A	4.1	rectangular	1	∞	0.041	
Res	1.0 <sup>(4)</sup>	rectangular	1	∞	0.01	
Temp	2.3 <sup>(5)</sup>	rectangular	1	∞	0.023	
R <sub>mis</sub>	2.0 <sup>(6)</sup>	normal	1	4	0.02	
U <sub>c</sub>	Combined standard uncertainty	normal		∞	0.071	
					expanded uncertainty (k=2)	14.2 %

**Note**

1. Preamplifier on, RBW 10 kHz
2. From Narda certificate
3. The contribution to the linearity is valid assuming measurements > 0.03 A/m (66dB Full Scale)
4. This value is calculated with detectable intensity magnetic field of 0.03 A/m. For larger values this contribution is negligible.
5. The temperature range is from 10 to 40°C.
6. This value is a reasonable one; it depend in reality to the environment condition and characteristics of the field source.

**- EHP200 Magnetic field – Mode B****Table IV.4 - Frequency range 300kHz- 30MHz <sup>(1)</sup> - Typical value of uncertainty**

Quantity	Relative standard uncertainty in (%)	Probability distribution	Sensitivity coefficient	Degrees of freedom	Relative uncertainty contribution (%)	
CF	3.9 <sup>(2)</sup>	normal	1	∞	0.039	
F	2.0	rectangular	1	∞	0.02	
L	2.7 <sup>(3)</sup>	rectangular	1	∞	0.027	
A	4.1	rectangular	1	∞	0.041	
Res	0.6 <sup>(4)</sup>	rectangular	1	∞	0.006	
Temp	2.3 <sup>(5)</sup>	rectangular	1	∞	0.023	
R <sub>mis</sub>	2.0 <sup>(6)</sup>	normal	1	4	0.02	
U <sub>c</sub>	Combined standard uncertainty	normal		∞	0.073	
					expanded uncertainty (k=2)	14.6%

**Note**

1. Preamplifier on, RBW 10 kHz
2. From Narda certificate
3. The contribution to the linearity is valid assuming measurements > 0,01 A/m (55.6dB Full Scale)
4. This value is calculated with detectable intensity magnetic field of 0.005 A/m. For larger values this contribution is negligible.
5. The temperature range is from 10 to 40°C
6. This value is a reasonable one; it depend in reality to the environment condition and characteristics of the field source.

**- EHP200 Electric field**
**Table IV.5 - Frequency range 9kHz- 27MHz <sup>(1)</sup> - Typical value of uncertainty**

Quantity	Relative standard uncertainty in (%)	Probability distribution	Sensitivity coefficient	Degrees of freedom	Relative uncertainty contribution (%)	
CF	3.9 <sup>(2)</sup>	normal	1	∞	0.039	
F	1.7	rectangular	1	∞	0.017	
L	2.7 <sup>(3)</sup>	rectangular	1	∞	0.027	
A	4.1	rectangular	1	∞	0.041	
Res	1.9 <sup>(4)</sup>	rectangular	1	∞	0.019	
Temp	2.3 <sup>(5)</sup>	rectangular	1	∞	0.023	
R <sub>mis</sub>	2.0 <sup>(6)</sup>	normal	1	4	0.02	
U <sub>c</sub>	Combined standard uncertainty	normal		∞	0.074	
					expanded uncertainty (k=2)	14.8%

**Table IV.6 - Frequency range 27MHz- 30MHz <sup>(1)</sup> - Typical value of uncertainty**

Quantity	Relative standard uncertainty in (%)	Probability distribution	Sensitivity coefficient	Degrees of freedom	Relative uncertainty contribution (%)	
CF	3.9 <sup>(2)</sup>	normal	1	∞	0.039	
F	8.6	rectangular	1	∞	0.086	
L	2.7 <sup>(3)</sup>	rectangular	1	∞	0.027	
A	4.5	rectangular	1	∞	0.045	
Res	1.9 <sup>(4)</sup>	rectangular	1	∞	0.019	
Temp	2.3 <sup>(5)</sup>	rectangular	1	∞	0.023	
R <sub>mis</sub>	2.0 <sup>(6)</sup>	normal	1	4	0.02	
U <sub>c</sub>	Combined standard uncertainty	normal		∞	0.114	
					expanded uncertainty (k=2)	22.8%

**Note**

1. Preamplifier on, RBW 10 kHz
2. From Narda certificate
3. The contribution to the linearity is valid assuming measurements > 0,2 V/m (60dB Full Scale)
4. This value is calculated with detectable intensity magnetic field of 0.15 V/m. For larger values this contribution is negligible.
5. The temperature range is from 10 to 40°C .
6. This value is a reasonable one; it depend in reality to the environment condition and characteristics of the field source.

In case is needed to measure more accurately, minimizing the measurement uncertainty, i.e. in case of experimental RF measurements, is possible to systematically correct the measures with the appropriate calibration factors, knowing the frequency of investigation or frequencies of investigations (i.e. harmonic components).

The reduction of the range of temperatures (i.e. from 18 to 30 ° C) allows to neglect the contribution due to temperature.

Moreover, knowing the range of intensities of the measured values, is possible (i.e. having a SIT certificate) to reduce the contribution of the non-linearity (obviously systematically correcting the measured value). Finally is possible to perform measurements on a single axis and thus make a negligible contribution due to the anisotropy.

In this way the total measurement uncertainty is reduced to the root mean square of the relative standard uncertainty of the calibration sensor  $u(CF)$  and the relative standard uncertainty of the repeatability  $u(R_{mis})$ :

$$U = \sqrt{u(CF)^2 + u(R_{mis})^2} .$$

## V. REFERENCES

- [1] "Guide to the expression of uncertainty in measurement" 1995 International Organization for Standardization.
- [2] L. Lo Presti, A De Marchi, *Inceteezze di misura*, CLUT, Torino, 1993
- [3] Athanasios Papoulis, "Probability, Random Variables and Stochastic Processes", McGraw-Hill, 1987